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I. INTRODUCTION

Integro-differential equations arise in many engineering and scientific disciplines, often as approximations to partial differential equations, which represent much of the continuum phenomena. Many forms of these equations are possible. In this report, we study identification of integro- differential equations, proposed to describe unsteady aerodynamics and aeroelasticity phenomena. The forms of equations studies in this report are closely related to the research of Tobak and Schiff [1-4] among others.

Let us consider the symmetrical longitudinal dynamics of an aircraft. In the linear quasi-steady regime, the aerodynamic pitch-moment coefficient, C_m , can be written as a linear combination of angle-of-attack, α , pitch rate, q , and elevator deflection, δ_e .

$$C_m = C_{m_o} + C_{m_\alpha}\alpha + \frac{c}{2V}C_{m_q}q + C_{m_{\delta_e}}\delta_e \tag{1}$$

The equation for the pitch coefficient implies that its values at any point in time depends only on the instantaneous values of α , q , and δ_e . This is a good approximation in quasi-steady flow. In regions where the flow field is highly unsteady, or if the elevator deflections are changed rapidly, it is reasonable to believe that the pitch moment coefficient depends on the past as well as the instantaneous values of the flow variables. Such conditions may occur in separated flow regions such as high angle-of- attack, stall, or spin. A reasonable extension to include past history of α in Equation (1) would lead to:

$$C_m = C_{m_o} + C_{m_\alpha}\alpha + \int_0^T F_{m_\alpha}(\tau)\alpha(t-\tau)d\tau + \frac{c}{2V}C_{m_q}q + C_{m_{\delta_e}}\delta_e \tag{2}$$

Of course, q and δ_e terms could be expanded similarly. Using the above equation for pitch movement coefficient, the pitch rate dynamics would be as follows:

$$I\dot{q} = q_\infty sc\{C_{m_o} + C_{m_\alpha}\alpha + \int_0^T F_{m_\alpha}(\tau)\alpha(t-\tau)d\tau + \frac{c}{2V}C_{m_q}q + C_{m_{\delta_e}}\delta_e\} \tag{3}$$

where :

- I : Pitch moment of inertia
- q : Dynamic Pressure
- s : Reference area
- c : Reference wing chord
- V : Aircraft Speed

We, thus, have the simplest form of integro-differential equation arising in unsteady aerodynamics. Similar results have been obtained by more systematic aerodynamic analyses. The integral term may also be written in terms of $\dot{\alpha}$ as it is usually done in aerodynamic analysis. Suppose we write (assuming $\alpha = 0$ for $t = -\infty$) :

$$C_m = C_{m_o} + \int_0^\infty H_{m_{\dot\alpha}}(\tau)\dot\alpha(t-\tau)d\tau + \frac{c}{2V}C_{m_q}q + C_{m_{\delta_e}}\delta_e \tag{4}$$

The form has been referred to as the indicial representation by aerodynamicists [3]. This equation is equivalent to Equation (2). Integrating the $\dot\alpha$ term by parts :

$$\int_0^\infty H_{m_{\dot\alpha}}(\tau)\dot\alpha(t-\tau)d\tau = -H_{m_{\dot\alpha}}(\tau)\alpha(t-\tau)|_o^\infty + \int_0^\infty \dot H_{m_{\dot\alpha}}(t-\tau)\alpha(t-\tau)d\tau$$

$$= H_{m_{\dot\alpha}}(0)\alpha(t) + \int_0^\infty \dot H_{m_{\dot\alpha}}(t-\tau)\alpha(t-\tau)d\tau. \tag{5}$$

Thus, Equation (4) is equivalent to Equation (3), if

$$H_{m_{\dot\alpha}}(0) = C_{m_\alpha}$$

and

$$\begin{aligned}\frac{d}{d\tau}H_{m_{\dot\alpha}} &= F_{m_\alpha} \quad 0 \le \tau \le T \\ &= 0 \qquad T < \tau\end{aligned} \tag{6}$$

It is reasonable to assume that in quasi-steady flow $H_{m_{\dot\alpha}(\tau)}$ will reach a constant value as $\tau \to \infty$. In practice, it is often convenient to use one form for identification and convert it into the other form for analysis and understanding of the aerodynamics phenomenon involved.

In unsteady aerodynamics and aeroelasticity, Equation (3) could be generalized in any of the following ways and possibly others.

(a) $C_{m_o}, C_{m_\alpha}, C_{m_q}$, and $C_{m_{\delta_e}}$ could be nonlinear functions of angle-of-attack, pitch rate and elevator deflection.
(b) The equation for C_m could involve double or higher order integrations, involving products of independent variables.
(c) Full six degree-of-freedom dynamics would bring in nonlinear kinematic terms.

This report addreses the following problems:

(a) Section 2 develops techniques for identifying integro- differential equations models from test or simulation data.
(b) Section 3 shows when the the integro-differential equation may be approximated by an ordinary differential equation.
(c) Conditions under which the integral term may be identified accurately are analyzed in Section 4.

The concepts developed in the previous sections will be applied to simulation data and high angle-of-attack test data in Section 5. Section 6 gives summary and conclusions of the results presented in this report.

II. IDENTIFICATION OF INTEGRO-DIFFERENTIAL EQUATIONS

The problem of estimating aircraft stability and control parameters has been researched extensively and many solutions have been proposed [6-8]. The identification of integro-differential equation models poses difficulties such that previous methods need to be extended. The discussion in this section will be limited to the form of Equation (3). As has been shown in the previous section, Equation (3) is equivalent to the form involving $\dot{\alpha}$ in the integral. Equation (3) is more suitable for identification, because α is usually measured while $\dot{\alpha}$ is not.

The identification problem for the pitch moment equation (3) involves the determination of (a) parameters $C_{m_o}, C_{m_\alpha}, C_{m_q}$ and $C_{m_{\delta_e}}$, (b) function $F_{m_\alpha}(\tau)$ $0 \leq \tau \leq T$, and (c) T. For the purpose of our discussion, we will assume that measurements $q(t), \alpha(t)$, and $\delta_e(t)$ are available from a flight test at sample points $t_k, k = 1, 2...N$. Other measurements can be included in the identification process without changing the procedure significantly.

The problem of identifying integro-differential equation models is, thus, different from the parameter estimation problem because it is necessary to identify a function in addition to parameters. There are two approaches to address the identification problem - identify the function directly, or approximate the function. In this section, we discuss each of the approaches.

2.1 Direct Identification of Integro-Differential Equations

To estimate the $F_{m_\alpha}(t)$ and the parameters, we consider Equation (3) and minimize the following performance index by choosing $C_{m_o}, C_{m_\alpha}, C_{m_q}$, and $C_{m_{\delta_e}}$, as well as $F_{m_\alpha}(\tau)$.

$$J = \frac{1}{2}\sum_{k=1}^{N}(q_m(k) - q(k))^2 \tag{7}$$

The first gradient of J with respect to $F_{m_\alpha}(\tau)$ is computed as follows:

$$g_F(t) = \frac{\partial J}{\partial F_{m_\alpha}} = -\sum_{k=1}^{N}(q_m(k) - q(k))\frac{\partial q}{\partial F_{m_\alpha}} \tag{8}$$

and

$$\frac{d}{dt}\{\frac{\partial q}{\partial F_{m_\alpha}}\} = \frac{q_\infty sc}{I}\{\alpha(t-\tau)\Delta + \frac{\partial q}{\partial F_{m_\alpha}}\} \tag{9}$$

where Δ represents a small region around τ over which $F_{m_\alpha}(\tau)$ is perturbed. Note that because $F_{m_\alpha}(\tau)$ is a continuous function, perturbation in $F_{m_\alpha}(\tau)$ at a single point will not change the value of q at all, since that will not change the integral. To perform any unique identification at all, it is necessary to assume that $F_{m_\alpha}(\tau)$ possesses certain continuity properties. This is in any case a good assumption for most physical systems.

Since α and q are highly nonlinear functions of $F_{m_\alpha}(\tau)$, an iterative procedure is needed to optimize the likelihood function. Either a first or second gradient procedure can be used. In a first gradient approach, we could start from a initial value of $F_{m_\alpha}(\tau)$ and update it as follows (k represents iteration number)

$$F_{m_\alpha}(\tau)|_{k+1} = F_{m_\alpha}(\tau)|_k \;\xi\; g_F(\tau)|_k \tag{10}$$

The value of ξ is selected based on a line search. The procedure can be repeated many times until convergence occurs. A second gradient approach could also be used.

The determination of the first gradient can be computationally time- consuming. Note that the first gradient is a function of τ and must be evaluated for all values of τ. That would require differential Equation (9) to be solved for all τ. Second gradient would be even more difficult to compute. Thus, it would be desirable from a computational point of view to divide up the interval between 0 and T into M intervals and estimate the values of $F_{m_\alpha}(\tau)$ at $M+1$ individual points, e.g.,

$$F_{m_\alpha}(i\frac{T}{M}), \qquad i = 0, 1, 2, ...M \tag{11}$$

This would require the solution to $M+1$ equation of the form of (6) to get the first gradient. To cut down the computation time, it is also possible to start with fewer intervals in early iteration and increase the number of intervals as the problem converges.

The problem of identifying T can be handled in one of two ways. One approach is to select a large enough T and estimate $F_{m_\alpha}(\tau)$. In principle, the estimated values of $F_{m_\alpha}(\tau)$ for τ greater than the true value of T should be small or zero. Else an estimate could be obtained by computing the gradients of the cost functional as follows:

$$g_T = \frac{\partial J}{\partial T} = -\sum_{k=1}^{N}(q_m(k) - q(k))\frac{\partial q(k)}{\partial T} \tag{12}$$

$$\frac{d}{dt}\{\frac{\partial q}{\partial T}\} = \frac{q_\infty sc}{I}\{F_{m_\alpha}(T)\alpha(t-T) + C_{m_q}\frac{\partial q}{\partial T}\} \tag{13}$$

The joint estimation of $C_{m_o}, C_{m_\alpha}, C_{m_q}$, and $C_{m_{\delta_e}}$, $F_{m_\alpha}(\tau)$, and T is a straight forward extension of the above procedure. If $F_{m_\alpha}(\tau)$ is estimated at many points, a first order gradient procedure may be the only viable option. For estimating a few points on the $F_{m_\alpha}(\tau)$ profile, a second gradient procedure is also usable.

2.2 Identification of Integro-Differential Equation Models by Approximation

Assumptions for Approximation

The identification problem is considerably simplified if the integral term is approximated by a sum of predefined functions, called basis functions. These approximations convert unknown functions into a small number of parameters, but require the following assumptions.

1. $F_{m_\alpha}(\tau)$ and other unknown functions have small high frequency components (note that since α multiplies $F_{m_\alpha}(\tau)$, the integral term in Equation (3), can be a major contributor to the pitching moment at high frequency).

2. $F_{m_\alpha}(\tau)$ and other unknown functions have finite tails; i.e., are zero for $\tau \geq T$.
3. $F_{m_\alpha}(\tau)$ is a continuous function, which converges quickly with any reasonable set of basis functions.

This identification procedure, therefore, consists of two steps - representation of indicial function $F_{m_\alpha}(\tau)$ by a set of basic functions and associated parameters, and then estimation of the resulting parametric models.

Representation of Integro-Differential Terms

Smooth functions have been parameterized in many different ways. For the purpose of identification, the following forms appear most appropriate.

(1) Polynomials: $F_{m_\alpha}(\tau)$ is written as a polynomial in τ

$$\begin{aligned} F_{m_\alpha}(\tau) &\approx A_0 + A_1\tau + A_2\tau^2 + \ldots\ldots, \quad && 0 \leq \tau \leq T \\ &\approx 0 && T < \tau \end{aligned} \tag{14}$$

A_0, A_1, A_2, A_3 are the unknown parameters. Numerical conditioning can be improved by either

(a) expanding the polynomial in , or

(b) using orthogonal polynomials.

(2) Splines: Linear, quadratic on cubic splines are excellent approximation to most continuous functions. Splines are polynomials whose coefficients change at certain break points, called knots [9]. The coefficients change in a constrained way to ensure continuity at knots. The unknown parameters are the polynomial coefficients as well as knot locations.

B-splines are preferred from a numerical viewpoint [9].

(3) Impulse Response of Rational Models: This form is useful when $T \to \infty$, since any other form requires too many parameters. The Laplace transform of $F_{m_\alpha}(\tau)$ could be written as a rational function of s

$$F_{m_\alpha}(s) = \frac{N(s)}{D(s)} \tag{15}$$

To ensure that $F_{m_\alpha}(\tau) \to 0$ as $\tau \to \infty$, $D(s)$ must have stable poles. For example, $F_{m_\alpha}(\tau)$ might be written as :

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} = \begin{pmatrix} -a_1 & 0 \\ 0 & -a_2 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \alpha \tag{16}$$

$$\int_{-\infty}^{t} F_{m_\alpha}(t-\tau)\alpha(\tau)d\tau \approx c_1\xi_1 + c_2\xi_2 \tag{17}$$

This will correspond to

$$\frac{N(s)}{D(s)} = \frac{b_1}{s+a_1} + \frac{b_2}{s+a_2} \tag{18}$$

This form has been used in aeroelastic studies.

(4) Steps: The indicial functions could be written as

$$\begin{aligned}
F_{m_\alpha}(\tau) &= a_1, \quad 0 \le \tau \le \frac{T}{n} \\
&= a_2, \quad \frac{T}{n} < \tau \le \frac{2T}{n} \\
&\vdots \\
&= a_n, \quad \frac{(n-1)T}{n} < \tau \le T \\
&= 0, \quad T < \tau
\end{aligned} \tag{19}$$

Identification Procedure

Once the integro-differential equation model is converted into a parametric form, previous estimation procedures can be used to determine the parameters as well as the model form [6]. The entire procedure is shown schematically in Figure 1.

III. APPROXIMATION OF INTEGRO - DIFFERENTIAL EQUATIONS

In this section, errors in approximating integro-differential equations by ordinary differential equations are estimated. The analysis addresses the specific class of equations encountered in unsteady aerodynamics and aeroelasticity.

Consider the integral term in Equation (2). Under certain conditions, this term is approximated as an add on to C_{m_α}. In the current analysis, we assume that the approximated form represents the entire pitch moment coefficient due to angle-of-attack.

$$\int_0^T F_{m_\alpha}(\tau)\alpha(t-\tau)d\tau = C_{m_\alpha}\alpha(t) \tag{20}$$

where C_{m_α} is a constant . For the two sides to equal in steady-state

$$\int_0^T F_{m_\alpha}(\tau)d\tau = C_{m_\alpha} \tag{21}$$

The approximation error is

$$\epsilon = \int_0^T F_{m_\alpha}(\tau)\alpha(t-\tau)d\tau - C_{m_\alpha}\alpha(t) \tag{22}$$

In the discussion of the approximation, we assume that $\alpha(t)$ is a continuous function with no jumps, such that $\dot{\alpha}(t)$ can be defined at each point. Since the secant slope cannot exceed the maximum tangential slope in an interval, we get :

$$||\alpha(t-\tau) - \alpha(t)|| \leq \tau||\dot{\alpha}_{max}(t)|| \tag{23}$$

where $\dot{\alpha}_{max}(t)$ is the maximum value of $\dot{\alpha}$ in the time interval from $t-\tau$ to t . Equation (22) therefore can be written as :

$$\begin{aligned}\epsilon &= \int_0^T F_{m_\alpha}(\tau)(\alpha(t-\tau) - \alpha(t))d\tau \\ &\leq \int_0^T ||F_{m_\alpha}(\tau)||(\alpha(t-\tau) - \alpha(t))||d\tau \\ &\leq \dot{\alpha}_{max}(t)\int_0^T ||F_{m_\alpha}(\tau)||\tau d\tau\end{aligned} \tag{24}$$

Thus, the maximum error depends on $\dot{\alpha}_{max}(t)$ and the first moment of $F_{m_\alpha}(\tau)$. If we make no assumption about the nature of $F_{m_\alpha}(\tau)$, then

$$\epsilon \leq T\dot{\alpha}_{max}(t)C_{m_\alpha} \quad : \quad General \tag{25}$$

For rectangular and triangular $F_{m_\alpha}(\tau)$, the approximation takes the forms

$$\epsilon \leq \frac{1}{2}T\dot{\alpha}_{max}(t)C_{m_\alpha} \quad : \quad rectangular$$

$$\epsilon \leq \frac{1}{3}T\dot{\alpha}_{max}(t)C_{m_\alpha} \quad : \quad triangular$$

Since the maximum approximated value of the integral term is $C_{m_\alpha}\alpha_{max}(t)$, the relative error becomes

$$\begin{aligned}
\epsilon_r &\leq T\frac{\dot{\alpha}_{max}(t)}{\alpha_{max}(t)} &&: \quad General\ C_{m_\alpha}(\tau) \\
&\leq \frac{1}{2}T\frac{\dot{\alpha}_{max}(t)}{\alpha_{max}(t)} &&: \quad rectangular \\
&\leq \frac{1}{3}T\frac{\dot{\alpha}_{max}(t)}{\alpha_{max}(t)} &&: \quad triangular
\end{aligned} \tag{26}$$

A similar result is obtained in approximating Equation (4) (Appendix A)

These error equations can be further simplified for improved understanding for motion at a single frequency ω. In that case

$$\dot{\alpha}_{max} \approx \omega\alpha_{max} \tag{27}$$

So the relative error may be written as :

$$\begin{aligned}
\epsilon_r &\leq \omega T &&: \quad General\ \ C_{m_\alpha}(\tau) \\
&\leq \frac{1}{2}\omega T &&: \quad rectangular \\
&\leq \frac{1}{3}\omega T &&: \quad triangular
\end{aligned}$$

The error introduced by the approximation increases with frequency of the motion and the maximum time delay for which the integral term is significant.

What do these two terms depend on? The frequency of the motion varies with applied inputs, natural short period dynamics and external disturbances. Most fixed-wing aircraft appear to have the natural frequency of the short period dynamics in the neighborhood of 0.5 Hz, though this number could be different for nonconventional aircraft. Pilot applied inputs are typically less than 1 Hz. Most turbulence spectra is also below 1 Hz.

Two factors can increase ω significantly. First, unsteady aerodynamics, vortex shedding and stall/spin can cause large rapid motions. Secondly, the aeroelastic phenomenon, because of high structural frequencies, almost always occurs at higher values of ω.

The time T can be physically thought of as the delay in reestablishing a new flow field following a change in one or more aerodynamic variables. Under quasi-steady conditions, T is proportional to the time taken by the flow field to travel a characteristic distance L. Thus

$$T = \frac{L}{V} \tag{28}$$

where V is aircraft speed. Thus T increases at low aircraft speed. If there is little or no interaction between the wing and the tail, the characteristic distance is the wing chord, while it could be as large as the aircraft length if vortex shedding at the nose can impact the tail.

Let us evaluate three situations to determine if the integral term can be important in evaluating pitching moment coefficient (in each case assuming the integral term is no worse than a triangular form).

Case 1: A fighter aircraft 60' long 10' wing chord, traveling at 800 ft /sec . Short period natural frequency is 0.5 Hz

$$T = \frac{10}{800} = \frac{1}{80}$$

$$\epsilon_r \leq \frac{1}{3} * 2 * \pi * .5 * \frac{1}{80} = .013$$

Case 2: The same aircraft in stall, nose/wing vortices hitting tail, forward speed 120 ft sec , forced dynamics at 0.75 Hz

$$\epsilon_r \leq \frac{1}{3} * 2 * \pi * .75 * \frac{60}{120} = .78$$

Case 3: The same aircraft with flutter behavior near 4 Hz. Forward speed 1000 ft sec . Unsteady aerodynamics requiring twice the normal time to settle down

$$\epsilon_r \leq \frac{1}{3} * 2 * \pi * 4 * 2\frac{10}{1000} = .17$$

The integral terms could be important in the last two cases. It has been conjectured by Tobak and Schiff [4] that in the quasi-steady state region at low angle-of-attack, the indicial terms are not important.

It should be noted that applying high frequency inputs does not necessarily make the indicial term more important. At high frequency, large amplitude inputs are needed to produce small motions in α and q . Thus, even though the indicial term is significant compared to $C_{m_\alpha}\alpha$, both of these terms could be small compared to the $C_{m_{\delta_e}}\delta_e$ term.

The main result of this section is therefore, as follows.

The integro-differential equations arising in unsteady aerodynamics and aeroelasticity may be approximated by differential equations when

$$\omega T << 1$$

where ω is the natural frequency of the motion and T is the time delay over which the indicial function is significant.

IV. IDENTIFIABILITY

To study the identifiability problem, we will start with a more general model for the pitch moment equation

$$I\dot{q} = q_\infty sc\{C_{m_o} + C_{m_\alpha}\alpha + \frac{c}{2V}C_{m_q}q + C_{m_{\delta_e}}\delta_e + \int_0^\infty F_{m_\alpha}\alpha(t-\tau)d\tau$$
$$+ \frac{c}{2V}\int_0^\infty F_{m_q}q(t-\tau)d\tau + \int_0^\infty F_{m_{\delta_e}}\delta_e(t-\tau)d\tau\} \tag{29}$$

The identifiability of the single-axis planar motion is analyzed to indicate some basic limitations to obtaining the values of functionals in integro-differential models. Let us assume that the $\alpha(t)$ and $\delta_e(t)$ can change independently. Correlation between $\alpha(t)$ and $\delta_e(t)$, caused by aircraft dynamics, will further reduce identifiability. It is also assumed that reference values of $q(t)$, and $\alpha(t)$ and $\delta_e(t)$ are selected such the C_{m_o} is zero.

Because of superposition, the identifiability of linear systems can be determined by considering a set of inputs at various frequencies and amplitudes. Let us write Equation (24) in the frequency domain

$$j\omega q(j\omega) = \frac{q_\infty sc}{I}\{C_{m_o} + C_{m_\alpha}\alpha(j\omega) + \frac{c}{2V}C_{m_q}q(j\omega) + C_{m_{\delta_e}}\delta_e(j\omega)$$
$$+ F_{m_\alpha}(j\omega)\alpha(j\omega) + \frac{c}{2V}F_{m_q}(j\omega)q(j\omega) + F_{m_{\delta_e}}(j\omega)\delta_e(j\omega)\} \tag{30}$$

We will apply sinusoidal inputs to the aircraft . Let

$$\alpha = \alpha_o e^{j\omega}$$
$$\delta_e = \delta_o e^{j\omega}$$

To evaluate identifiability, we can select two linearly independent combinations of α_o and δ_o , e.g.,

$$\alpha_o = 1\ ,\ \delta_o = 0;\ and$$
$$\alpha_o = 0\ ,\ \delta_o = 1. \tag{31}$$

Because of linearity, other values of α_o and δ_o do not affect system identifiability even though they lead to reduced estimation errors in parameters which are identifiable. Thus at any ω there are four scalar equations (two complex equations) and nine unknowns (C_{m_q}, C_{m_α},$C_{m_{\delta_e}}$and real and imaginary parts of F_{m_α} , F_{m_q} and $F_{m_{\delta_e}}$. The parameters C_{m_α}, C_{m_q}, $C_{m_{\delta_e}}$are not functions of frequency.

At zero frequency, we have two equations and three unknowns (C_{m_α}, C_{m_q}, $C_{m_{\delta_e}}$). The zero frequency input data provides a mechanism to solve for $C_{m_\alpha}$and C_{m_q}in terms of $C_{m_{\delta_e}}$ (or any two variables in terms of the third). Thus, at any frequency there are four equations and seven unknowns. Without further assumptions, linear indicial function representation in the pitch plane is not completely identifiable from flight test data.

The inclusion of C_{m_o} in terms of $C_{m_{\delta_e}}$ by including a test point at $\alpha_o = 0$ and $\delta_o = 0$ in addition to the two test points described by Equation (26).

The non-identifiability of $F_{m_q}(t)$, $F_{m_\alpha}(t)$, and $F_{m_{\delta_e}}(t)$ indicates that further constraints are needed on the general nature of these functions. Some of the properties which should hold because of the basic nature of fluid flow can be used to bring about identifiability. These properties relate to smoothness condition discussed in Section 2.

V. SIMULATION AND FLIGHT TEST RESULTS

Many integro-differential models have been identified using the approaches discussed in previous sections. This section shows results obtained from a simulation model. The goals of the presentation is to illustrate the theoretical results. Finally, a flight test data set is studied in the next section to qualitatively evaluate the need for indicial terms.

5.1 Simulation Data

The simulated model is of the form

$$\dot{\alpha} = Z_\alpha \alpha + q$$
$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta_e} \delta_e + \int_0^T M_q(\tau) q(t-\tau) d\tau \tag{32}$$

A dimensional model has been selected to eliminate various scaling parameters. The following values are used in simulations, throughout

$$Z_\alpha = -2 \ , \ M_\alpha = -5 \ , \ M_q = -2 \ , \ M_{\delta_e} = -5 \tag{33}$$

Note that we have selected an indicial term in q , a linear combination of $\dot{\alpha}$ and α. Of course, this term can also be written in terms of α, using integration by parts. The characteristic equation of this model is $s^2 + 4s + 9$. So the natural frequency is 3 rad sec (about $\frac{1}{2}$ Hz).

In our simulations, we chose two values of T, .5 and .05. ωT then is 1.5 and .15.

$\omega T = 1.5$

The modeled form of $M_q(t)$ is shown in Figure 2. The simulation model was developed rapidly using System_Build , a feature of MATRIX_X [10]. The block diagram is shown in Figure 3. An input time history shown in Figure 4 is applied to give an α , q , $\dot{q}$ response of Figure 5.

At first least squares estimation approach is used and $\dot{q}$ is derived by differentiating q (a simple two point differentiating is used throughout). The resulting model without noise is shown in Figure 6. Almost all of the error is due to uncertainties in obtaining $\dot{q}$ from q. The estimated parameter values are exact if simulation values of $\dot{q}$ is used directly in estimation.

Figure 7 shows the effect of adding noise to q on model estimation accuracy. Table 1 shows theoretical values of estimation error for random noise in $\dot{q}$. These results indicate that the least squares approach does not give an acceptable estimate for indicial models unless a more complex input is used. The least squares method nevertheless is good for selecting significant terms.

The approximation to this indicial model would be as follows.

$$\dot{q} = -5\alpha - 2q - 5\delta_e - 2.1q \tag{34}$$

When an attempt is made to identify this model, the estimated parameters are shown in Table 2 and the fit error to $\dot{q}$ is shown in Figure 8.

Maximum likelihood technique is used to estimate the indicial model from noisy measurements of angle-of-attack, α, and pitch rate q. The estimated indicial function is shown in Figure 9, and the fit to α and q time history is shown in Figure 10.

$\omega T = 0.15$

The indicial function is now spread out over the first .05 sec, but is scaled up by a factor of 10 from Figure 2, such that the approximation is the same as in Equation (34). The input is the same as before. The responses are shown in Figure 11. The following steps are repeated.

(1) $\dot{q}$ is obtained by differentiating q and the indicial model is identified. The results are shown in Figure 12 and are much poorer than before.

(2) The estimates are so poor without noise that very little noise causes the estimates to lose all accuracy.

(3) Table 3 shows that the simplified model may be identified better than for $\omega T = 1.5$. Figure 13 shows fit errors in $\dot{q}$.

(4) Application of maximum likelihood approach does not aid the estimation of the indicial function.

5.2 Flight Data

This section shows data from a spin research vehicle (SRV) and demonstrates the need to use indicial function representations in the high angle-of-attack region. The 3/8-scaled unpowered model of a high performance fighter aircraft is dropped from a B-52 and is controlled remotely. The vehicle provides an effective means to conduct aerodynamic tests in post-stall and spin regions. Though the flight test lasts several minutes, a 40-second long segment shown in Figure 14 is studied. A standard set of on-board instrumentation is available including (3-axis rate and attitude gyros and 3-axis accelerometer, dynamic pressure, angle-of-attack, sideslip angle, static pressure, and control position).

We will look at data qualitatively in this report to show the need for integro-differential function models or a function with memory. Figure 15 shows the normal force coefficient time history as a function of angle-of- attack. Based on known aircraft behavior, the lift coefficient depends primarily on angle-of-attack and to a lesser extent on elevator deflection and other aerodynamic variables. The dependence of C_N on variables other than the angle-of-attack cannot account for the large loops. Note that for fixed controls between 28s and 40s, the aircraft exhibits oscillatory behavior with increasing angle-of- attack. This is an interesting region to analyze because forces and moments are affected only by aerodynamic variables.

Figure 15 indicates that the phenomenon is not aerodynamic hysterisis, because in hysterisis the function follows one path for increasing and another one for decreasing values of the independent variable. The behavior of the aircraft for angle-of-attack between $70°$ and $80°$ is qualitatively very interesting. It is known from linearized analysis [8] that over 40° angle-of-attack, the magnitude of C_{N_α} decreases with angle-of-attack. In the time history shown in Figure 16 the angle-of-attack suddenly jumps from about $40°$ to $80°$ and the lift stays high (corresponding more to the $40°$ lift coefficient than the $80°$ lift coefficient). Then about 0.5 sec later the lift coefficient drops suddenly. Physcially, this may occur because a sudden increase in angle-of-attack causes the flow to remain attached for a short period of time and the lift increases dramatically. The flow must eventually separate causing the lift to drop. The phenomenon may be related to what has been referred to as "dynamic lift".

The plots also show that the natural frequency is about 0.5 Hz. Thus an indicial function with 0.5 sec. delay should be clearly identifiable. The identified model had been shown previously as Figure 17 in Reference [5].

Note that for responses below 40° angle-of-attack, the indicial model is not identifiable and the indicial term may be approximated by a lumped coefficient.

VI. SUMMARY AND CONCLUSIONS

The report presented approaches to identify intergro-differential equation models which arise in aeroelastic and unsteady aerodynamics. The following conclusions may be drawn.

1. When the product of frequency of motion and maximum time delay is much smaller than one, the integral term can be approximated by a constant. When this product is of the order of, or larger than one, the integral term cannot be approximated.
2. Integro-differential models are in general non-identifiable. Approximations are needed to bring about identifiability.
3. Least-squares method may be used for model determination but the maximum likelihood technique is needed to accurately estimate parameters.
4. High angle-of-attack and post-stall/spin region appears to have characteristics, which can be satisfied by indicial models.

More work though is needed to advance better understanding of unsteady- aerodynamics and aeroelastic phenomena from measured data.

Appendix A

APPROXIMATION OF $\int_0^\infty H_{m_{\dot\alpha}}(\tau)\dot\alpha(t-\tau)d\tau$

We will attempt to approximate this integral by C_{m_α} . For this approxiamtion to hold in steady state

$$C_{m_\alpha}(t) = H_{m_{\dot\alpha}}(\infty) \tag{A.1}$$

The error is approximated as follows:

$$\begin{aligned}\epsilon &= \int_0^\infty H_{m_{\dot\alpha}}(\tau)\dot\alpha(t-\tau)d\tau - C_{m_\alpha}\alpha(t)\\ &= \int_0^\infty \{H_{m_{\dot\alpha}}(\tau) - H_{m_{\dot\alpha}}(\infty)\}\dot\alpha(t-\tau)d\tau\end{aligned} \tag{A.2}$$

It is reasonable to assume that $H_{m_{\dot\alpha}}(\tau)$ reaches its steady-state value $H_{m_{\dot\alpha}}(\infty)$ for $\tau = T$. Then

$$\begin{aligned}|H_{m_{\dot\alpha}}(\tau) - H_{m_{\dot\alpha}}(\infty)| &\leq (T-\tau)|\dot H_{m_{\dot\alpha}}|_{max} \quad && \tau \leq T\\ &\approx 0 && \tau > T\end{aligned} \tag{A.3}$$

Then (A.2) becomes

$$\epsilon \leq c|\dot H_{m_{\dot\alpha}}|_{max}\dot\alpha_{max}T^2 \tag{A.4}$$

where c is a constant. Since $\dot H_{m_{\dot\alpha}}$ is the same as $F_{m_\alpha}(\tau)$ and the integral of $F_{m_\alpha}(\tau)$ from 0 to T is C_{m_α}, Equation (A.4) becomes

$$\epsilon \leq c' C_{m_\alpha}\dot\alpha_{max}T \tag{A.5}$$

Equations can also be developed to approximate the integral term by a combination of α and $\dot\alpha$ coefficients.

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TABLE 1: COMPUTED ESTIMATION ERROR FOR LEAST SQUARES
(500 Points, RMS Noise in q = .001)

Parameter	Value	Estimation Error
M_α	-5.0	.0048
M_q	-2.0	.0167
M_{δ_e}	-5.0	.0045
M_o	-0.0	.00004
$M_q(.1)$	-0.3	.0253
$M_q(.2)$	-0.6	.0213
$M_q(.3)$	-0.7	.0183
$M_q(.4)$	-0.4	.0128
$M_q(.5)$	-0.2	.0045

TABLE 2: ESTIMATED PARAMETERS IN REDUCED MODEL ($\omega T = 1.5$)

Parameters	Values	Estimated Value (No noise)
M_α	-5.0	-9.75
$M_q(equi.)$	-4.1	-3.79
M_{δ_e}	-5.0	-6.28
M_o	0.0	+0.0

TABLE 3: ESTIMATED PARAMETERS IN REDUCED MODEL ($\omega T = .15$)

Parameters	Values	Estimated Value (No noise)
M_α	-5.0	-5.3
$M_q(equi.)$	-4.1	-4.4
M_{δ_e}	-5.0	-5.3
M_o	0.0	+0.0

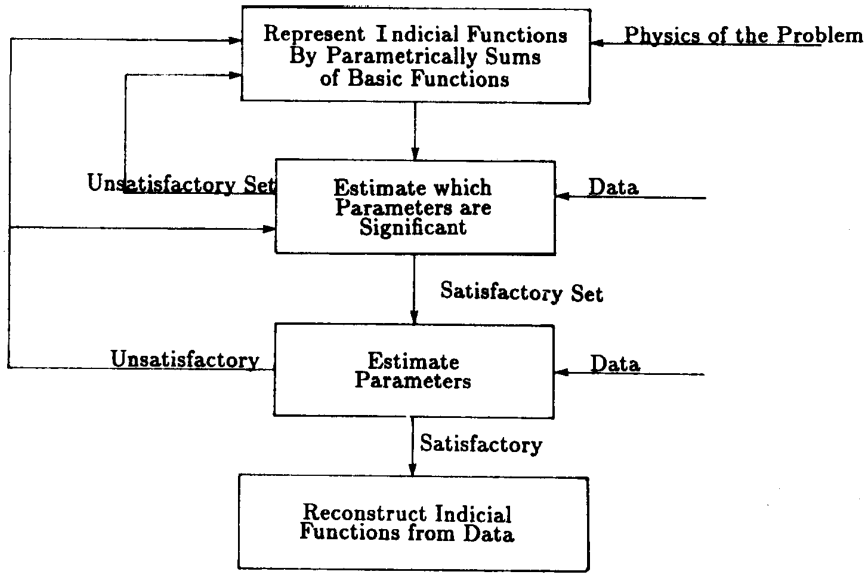

Figure 1 : Schematic Procedure for the Estimation of Integro- Differential Models.

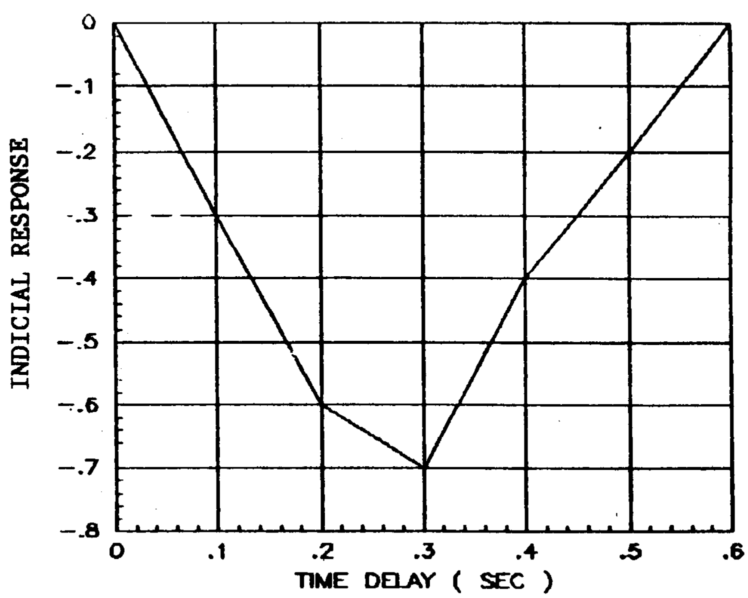

Figure 2 : Indicial Models Used in Simulation

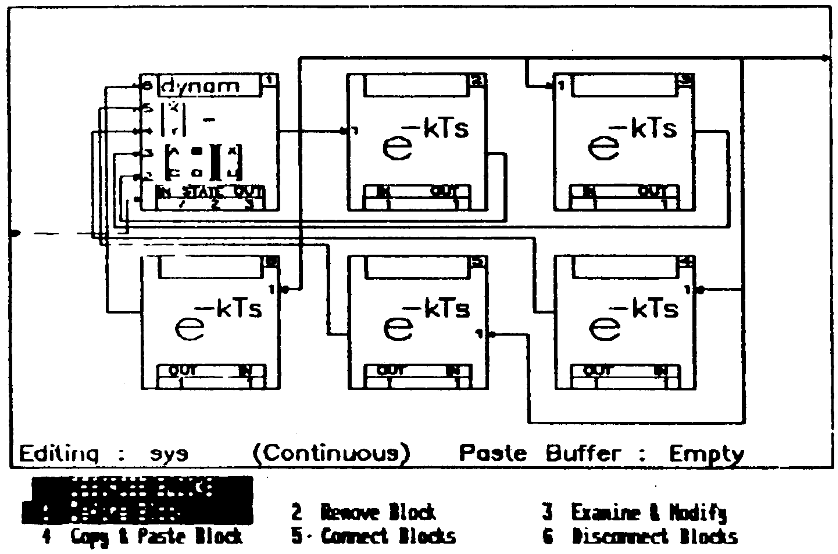

Figure 3 : MATRIX$_X$ Block-Diagram Model of the Integro-Differential Model

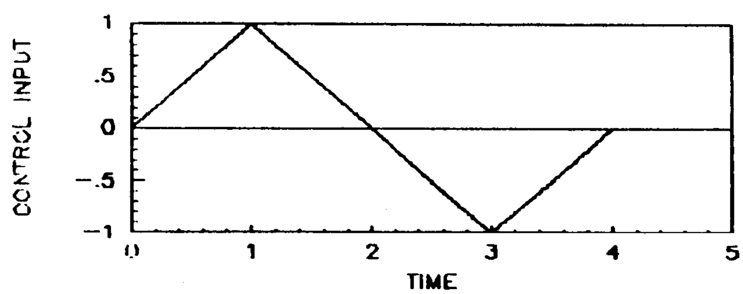

Figure 4 : Time History of Applied Input

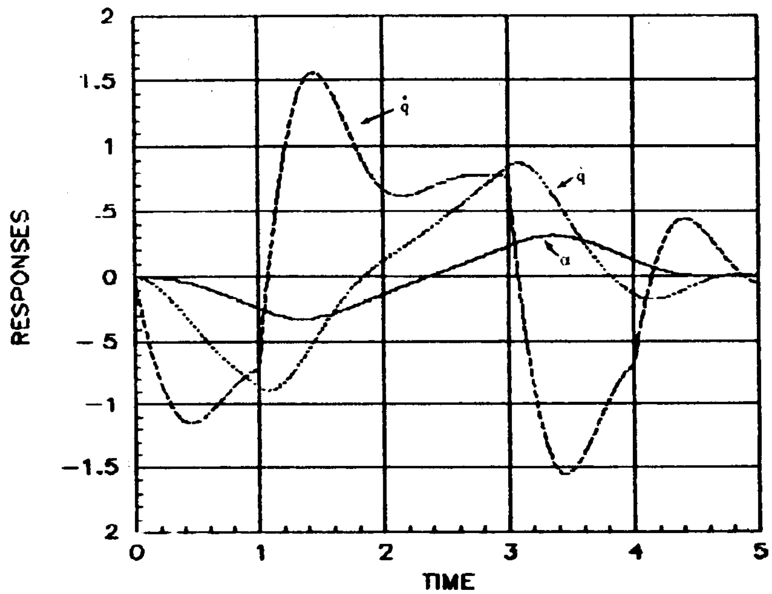

Figure 5 : Response of Indicial Model to Input of Figure 4.

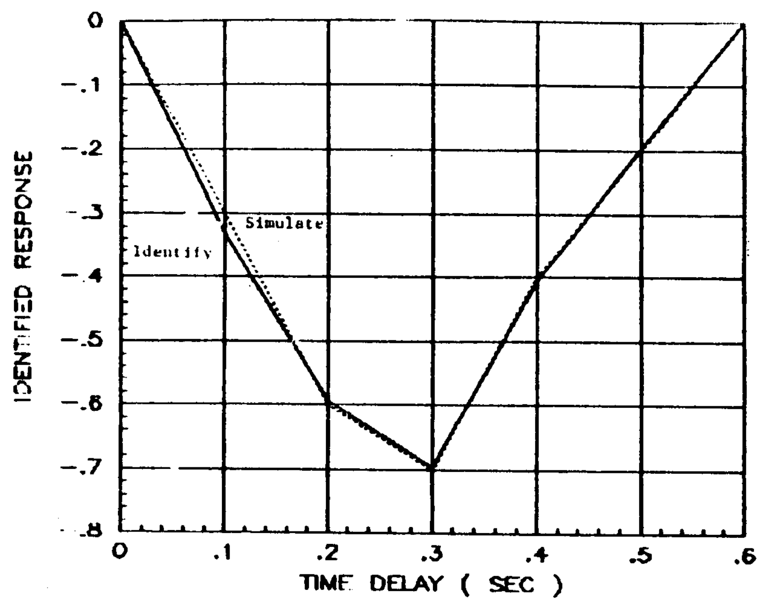

Figure 6 : Comparison of Modeled and Simulated Response

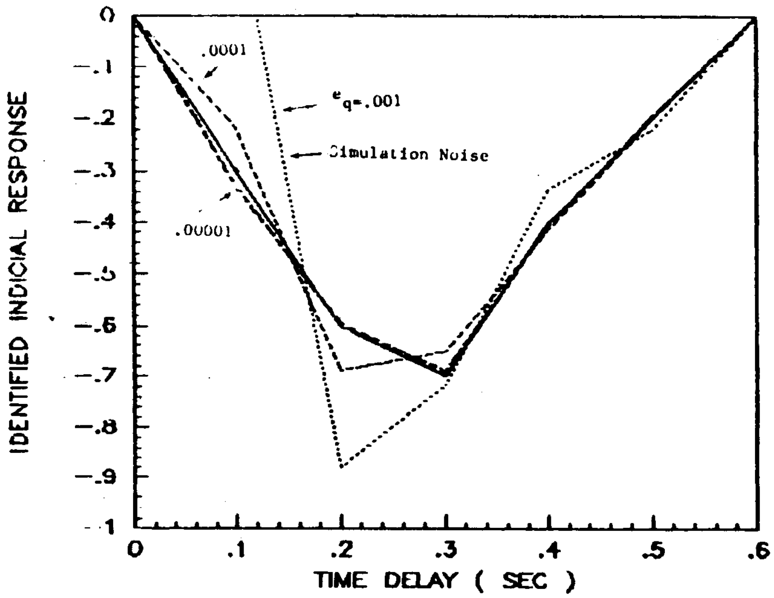

Figure 7 : Effect of Noise in q on Least Squares Estimation Accuracy

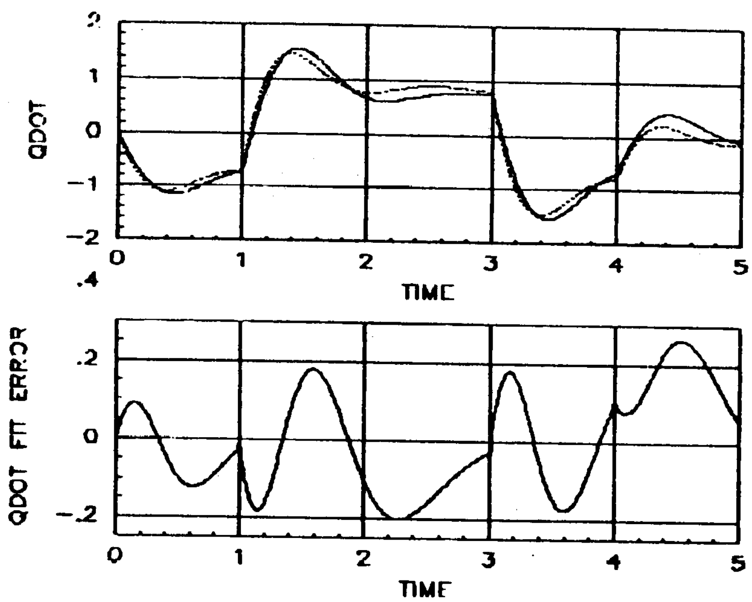

Figure 8 : Fit-Error in q when an Indicial Model is not Used in Estimation ($\omega T = 1.5$)

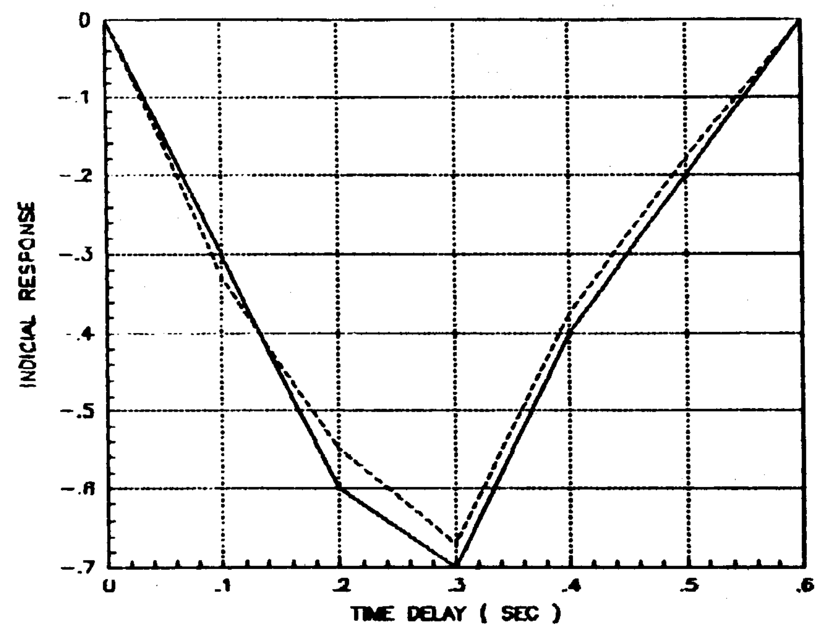

Figure 9 : Indicial Functions Identified With Maximum Likelihood $(\omega T = 1.5)$

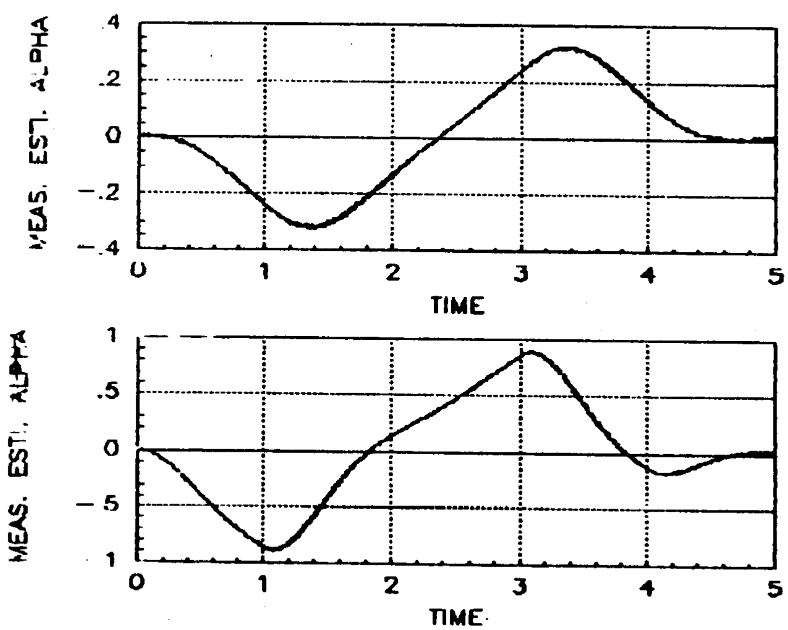

Figure 10 : Identified and Simulated Time Histories Using Maximum- Likelihood $(\omega T = 1.5)$

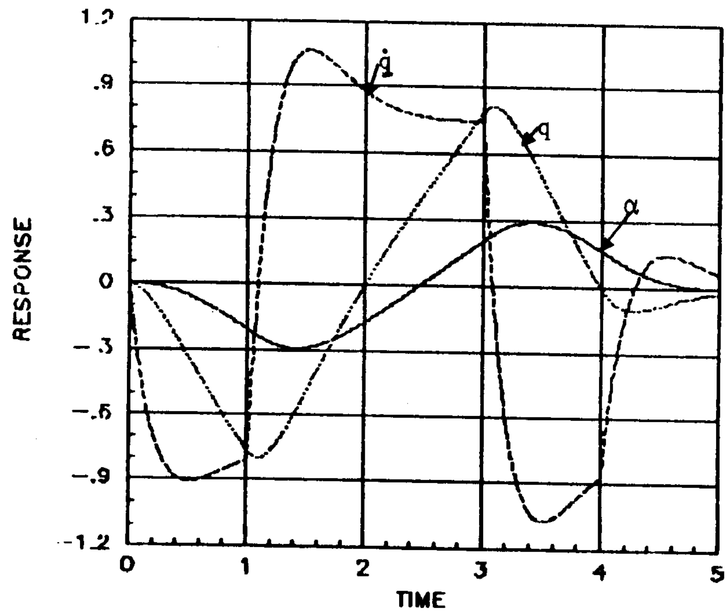

Figure 11 : System Response for $\omega T = .15$

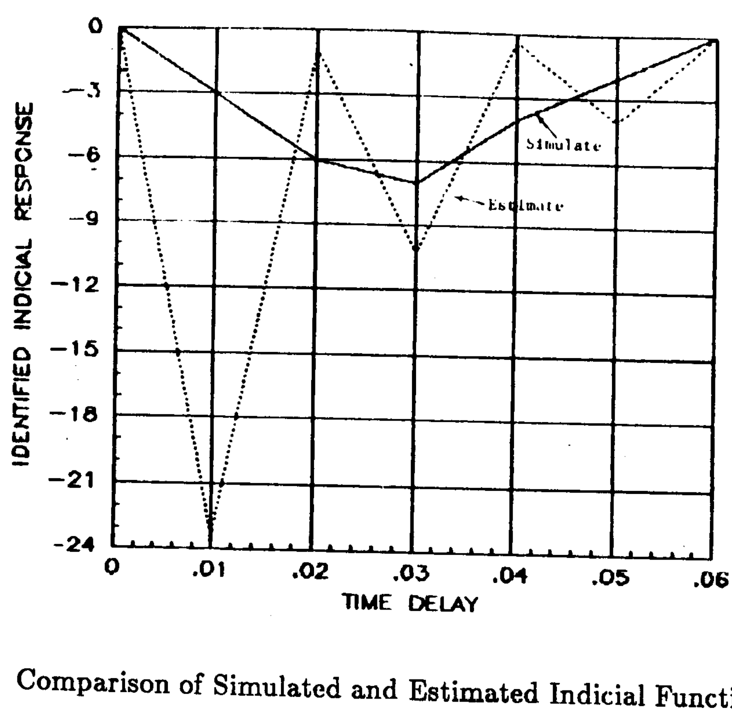

Figure 12 : Comparison of Simulated and Estimated Indicial Function for $\omega T = .15$.

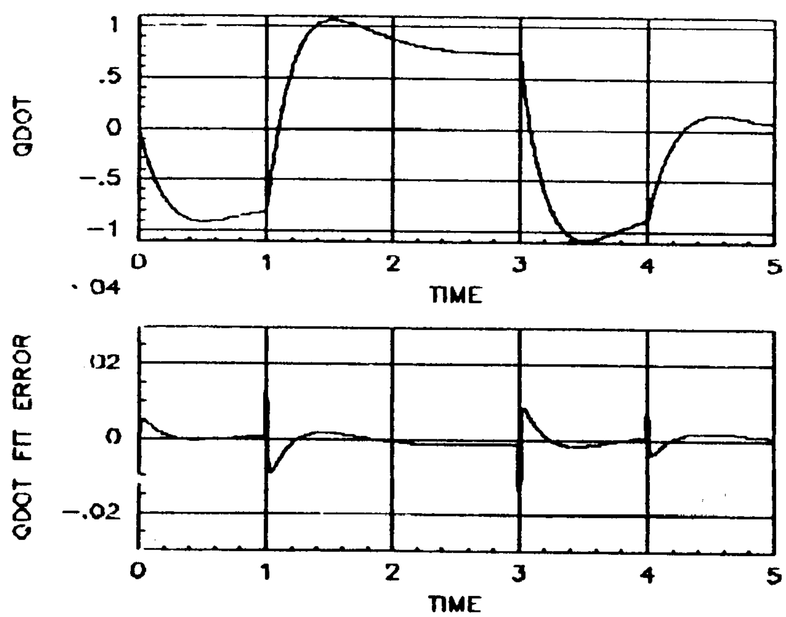

Figure 13 : $\dot{q}$ Fit Using Least- Squares and Reduced Model.

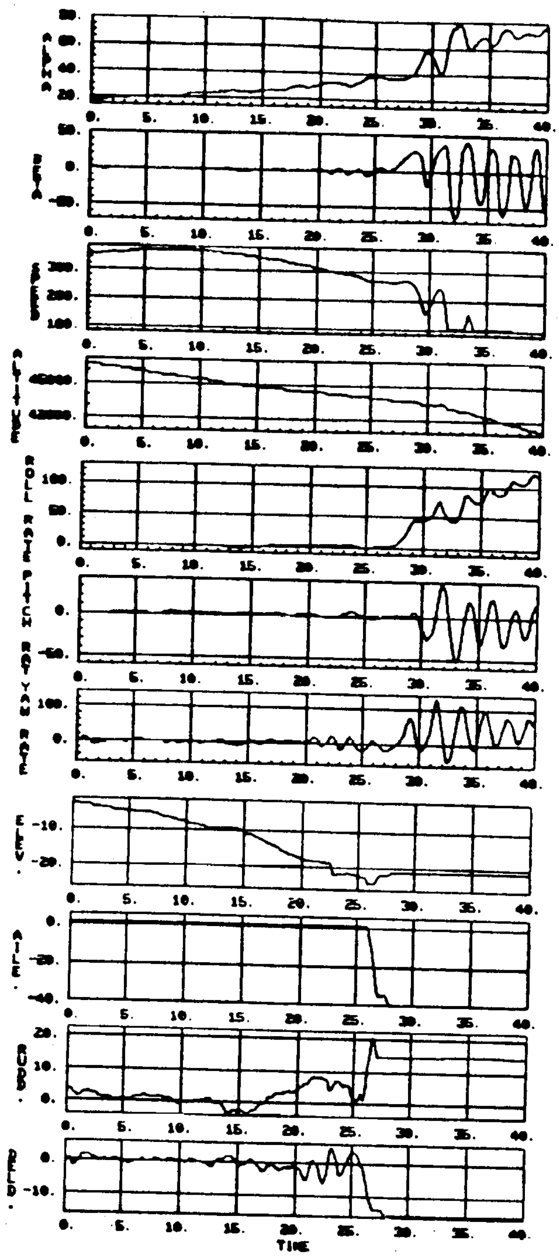

Figure 14 : Aircraft Flight Data (All data in ft. , sec. , deg.)

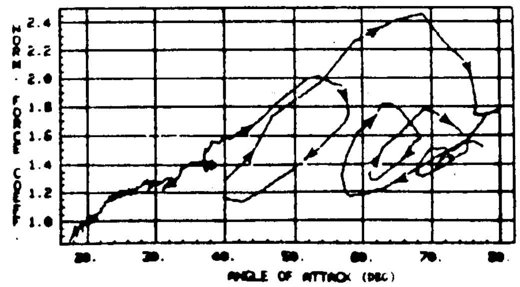

Figure 15 : Normal Force Coefficient as a Function of Angle-of-Attack

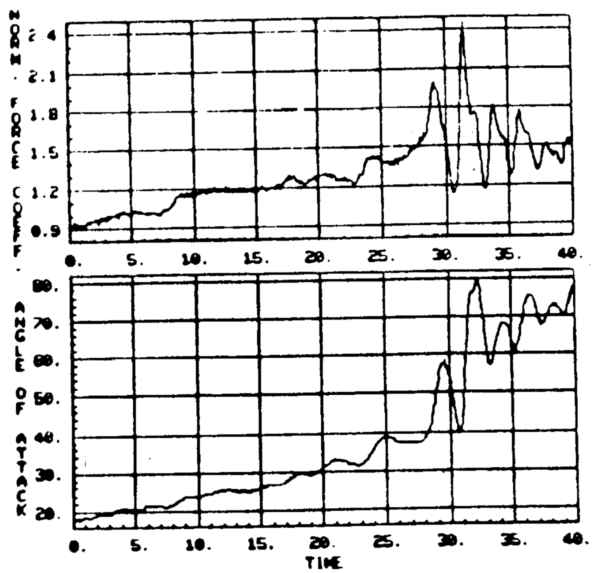

Figure 16 : Normal Force Coefficient and Angle-of Attack Time Histories

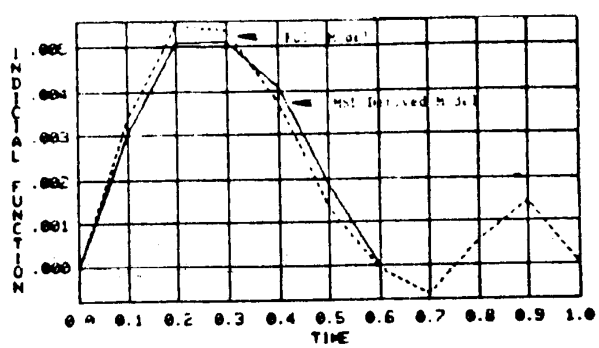

Figure 17 : Estimated Indicial Function

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16. Abstract

This report deals with the problem of estimating integro-differential models based on test or simulation data. The identification techniques proposed for estimating parameters in models described by differential equations need to be considerably extended to deal with the integral terms. Conditions under which the integral terms may be approximated by algebraic values are discussed. The integro-differential models discussed in this report are related to indicial models proposed by aerodynamicists to describe unsteady flow.

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